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# Fault-Tolerant Control of Large Space Structures Using the Stable Factorization Approach

H. C. Razavi, R. K. Mehra,  
and M. Vidyasagar

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# Fault-Tolerant Control of Large Space Structures Using the Stable Factorization Approach

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## FOREWORD

This report describes the work performed by Scientific Systems, Inc. from 12/12/84 to 7/10/85 under National Aeronautics and Space Administration Award No. NAS1-17946. The work represents the first of a three-phase project aimed at development of theory, computational techniques, and a commercial grade control synthesis/analysis software package for large space structures; the approach is based on recent stable factorization and  $H_\infty$  techniques. The objective of Phase I effort was to demonstrate the feasibility of the concept. Dr. Earnest Armstrong was the project director for NASA.

Dr. Hamid Razavi was the principal investigator. Professor M. Vidyasagar of the University of Waterloo was the consultant. Special thanks go to Ms. Alina Bernat for supervising the documentation.

## SUMMARY

Large space structures are characterized by the features that (i) they are in general infinite-dimensional systems, and (ii) they have large numbers of undamped or lightly damped poles. Any attempt to apply linear control theory to large space structures must therefore take into account these features. Phase I of this project consisted of an attempt to apply the recently developed Stable Factorization (SF) design philosophy to problems of large space structures, with particular attention to the aspects of robustness and fault tolerance.

The final report on the Phase I effort consists of four sections, each devoted to one task. The first three sections report new theoretical results, while the last consists of a design example. Significant new results were obtained in all four tasks of the project. More specifically, (a) an innovative approach to order reduction was obtained, (b) stabilizing controller structures for plants with an infinite number of unstable poles were determined under some conditions, (c) conditions for simultaneous stabilizability of an infinite number of plants were explored, and (d) a fault-tolerant controller design that stabilizes a flexible structure model was obtained which is robust against one failure condition.

The overall objective of Phase II effort will be to further develop the analytical foundation laid in Phase I, evolve computational algorithms, and test the stable factorization methodology on a realistic model of a large flexible structure. Thus by the end of Phase II effort the stage will be set for the development (in Phase III) of a state-of-the-art computer-aided control system synthesis/analysis package for large space structures, based on the stable factorization/ $H_\infty$  approach.

## 0. INTRODUCTION

In Phase I of this project, we proposed four tasks, namely (i) order reduction, (ii) coprime factorizations, (iii) robustness, and (iv) applications. Of these, the first task was carried out to its natural completion, while the progress realized on the other three was more than sufficient to demonstrate their feasibility.

We now describe in detail the four Phase I tasks and the results obtained.

Throughout this report, the following symbols are used:  $S$  denotes the set of proper stable rational functions, i.e. functions that are proper and have all of their poles in the open left half-plane.  $A$  denotes the set of distributions  $f(\cdot)$  of the form

$$f(t) = \sum_{i=0}^{\infty} f_i \delta(t-t_i) + f_a(t), \quad (0.1)$$

where  $\delta$  denotes the unit impulse distribution,  $0 \leq t_0 < t_1 < \dots$ ,  $f_a(\cdot)$  is a Lebesgue measurable function, and in addition,

$$\|f\|_A = \sum_{i=0}^{\infty} |f_i| + \int_0^{\infty} |f_a(t)| dt \quad (0.2)$$

is finite. One can think of  $A$  as the set of all regular measures of bounded variation on the half-line  $[0, \infty)$  that do not have any singular part; see Rudin (1962) or Hille and Phillips (1957). We refer to  $f_a$  as the nonatomic part of  $f$ , and to the sum of delayed impulses as the atomic part

of  $f$ . The set  $\hat{A}$  consists of all Laplace transforms  $\hat{f}$  of distributions in  $f$ , defined according to the familiar rule.

$$\hat{f}(s) = \sum_{i=0}^{\infty} f_i e^{-st_i} + \int_0^{\infty} f_a(t) e^{-st} dt. \quad (0.3)$$

If  $\hat{f} \in \hat{A}$ , we define  $\|\hat{f}\|_{\hat{A}} = \|f\|_A$  where  $f$  is the inverse Laplace transform of  $\hat{f}$ . Note that  $S$  is a subset of  $\hat{A}$ , and that  $S$ ,  $A$ ,  $\hat{A}$  are all rings. We use the symbols  $M(S)$ ,  $M(A)$  and  $M(\hat{A})$  to denote the set of matrices, of whatever order, whose elements all belong to  $S$ ,  $A$ ,  $\hat{A}$ , respectively.

On the set  $\hat{A}$  it is sometimes convenient to use the so-called  $H_{\infty}$ -norm, denoted by  $\|\cdot\|_{\infty}$ , and defined by

$$\|\hat{f}\|_{\infty} = \sup_{\omega} |\hat{f}(j\omega)| = \sup_{\text{Re } s > 0} |\hat{f}(s)|. \quad (0.4)$$

Note that

$$\|\hat{f}\|_{\infty} \leq \|\hat{f}\|_{\hat{A}} \text{ for all } \hat{f} \in \hat{A}. \quad (0.5)$$

Finally, we introduce the so-called disk algebra, denoted by  $A_D$ , which consists of functions that are analytic on the open unit disk and continuous on the closed unit disk. The norm on  $A_D$  is defined by

$$\|f\|_{A_D} = \max_{|z| \leq 1} |f(z)| = \max_{\theta \in [0, 2\pi]} |f(e^{j\theta})|. \quad (0.6)$$

# 1. TASK 1: ORDER REDUCTION

Task 1 was as follows: find computational algorithms for the reduced-order modeling of large or infinite-dimensional systems, where the objective is to minimize the graph metric distance between the true system and the reduced-order model.

For ease of discussion, we reproduce below the definition and some properties of the graph metric, as taken from Vidyasagar (1984), Vidyasagar (1985), and Vidyasagar and Kimura (1984). Given a matrix  $P$ , a pair  $(N, D)$  is said to be a right-coprime factorization (r.c.f.) of  $P$  if (i)  $N, D \in M(\mathbb{A})$ , (ii)  $P(s) = N(s)[D(s)]^{-1}$  such that

$$X(s) N(s) + Y(s) D(s) = I \text{ for all } s. \quad (1.1)$$

If, in addition, it is true that

$$N^*(j\omega) N(j\omega) + D^*(j\omega) D(j\omega) = I \text{ for all } \omega, \quad (1.2)$$

where  $*$  denotes the conjugate transpose, then  $(N, D)$  is said to be normalized r.c.f. or n.r.c.f. of  $P$ . If  $P$  has an r.c.f., then it has an n.r.c.f.; see Sz. Nagy and Foias 1970. Note that  $(N, D)$  is an n.r.c.f. if and only if the matrix

$$B = \begin{bmatrix} N \\ D \end{bmatrix} \quad (1.3)$$

is inner. If  $P$  has an n.r.c.f., then it is unique to within right multiplication by an orthogonal matrix.

Next, we define the graph metric. Let  $P_1, P_2$  be plants of the same dimensions, and let  $(N_1, D_1), (N_2, D_2)$  be n.r.c.f.'s of  $P_1$  and  $P_2$ , respectively. Define inner matrices  $B_1, B_2$  as in (3.1.3), and let

$$\delta(P_1, P_2) = \inf_{U \in M(A), \|U\|_\infty \leq 1} \|B_1 - B_2 U\|_\infty \quad (1.4)$$

Then the graph metric distance  $d(P_1, P_2)$  between the two plants is defined by

$$d(P_1, P_2) = \max \{ \delta(P_1, P_2), \delta(P_2, P_1) \}. \quad (1.5)$$

Now we give some criteria for the robustness of feedback stability, given a plant  $P$  and a controller  $C$  of commensurate dimensions, define

$$\begin{aligned} T(P, C) &= \begin{bmatrix} (I+PC)^{-1} & -P(I+CP)^{-1} \\ C(I+PC)^{-1} & -CP(I+CP)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} I \\ C \end{bmatrix} (I+PC)^{-1} [I \quad -P]. \end{aligned} \quad (1.6)$$

Suppose a nominal plant-compensator pair  $(P_0, C_0)$  is stable, and suppose  $P_0, C_0$  are perturbed to  $P, C$  respectively. Then the pair  $(P, C)$  is also stable provided

$$d(P, P_0) \|T(P_0, C_0)\|_\infty + d(C, C_0) \|T(C_0, P_0)\|_\infty < 1. \quad (1.7)$$

In particular,  $C_0$  stabilizes  $P$  provided

$$d(P, P_0) \|T(P_0, C_0)\|_\infty < 1. \quad (1.8)$$

This concludes the summary of known results, and also shows why the graph metric is a good measure of proximity in comparing unstable or nearly unstable systems. Note that there are no assumptions to the effect that  $P, P_0$  must have the same number of RHP poles, or that they should have no  $j\omega$ -axis poles, as is the case in the work of Doyle and Stein 1981. Thus

the above theory is ideally suited for studying the robustness of feedback stability in situations where the plant poles migrate across the stability boundary as the plant parameters vary.

Now consider the problem of order reduction in the graph metric. Suppose  $P_t$  is a "true" plant, for which we wish to construct an "approximate" model  $P_a$ . Then  $P_a$  is used to design a controller  $C$ , designed on the basis of the approximate model  $P_a$ , to stabilize the true plant  $P_t$ . We know this is the case provided

$$d(P_a, P_t) < 1/\|T(P_a, C)\|_\infty. \quad (1.9)$$

Next, suppose  $(N_t, D_t)$  is an n.r.c.f. for  $P_t$ , and  $(N_a, D_a)$  is a (not necessarily normalized) r.c.f. of  $P_a$ . Define

$$B_t = \begin{bmatrix} N_t \\ D_t \end{bmatrix}, \quad M_a = \begin{bmatrix} N_a \\ D_a \end{bmatrix}, \quad (1.10)$$

$$v = \|B_t - M_a\|_\infty. \quad (1.11)$$

It is shown in Vidyasagar 1984 that if  $v < 1$ , then

$$d(P_a, P_t) \leq \frac{2v}{1-v}, \quad (1.12)$$

In fact, a close examination of the proof reveals that  $(N_a, D_a)$  need only be a right fractional representation of  $P_a$ ; it need not be coprime. This suggests the following procedure for approximating  $P_t$ .

- (i) Find a normalized r.c.f.  $(N_t, D_t)$  for  $P_t$ .
- (ii) Form the matrix  $B_t$ .
- (iii) Approximate  $B_t$  by  $M_a$ .

To date, there are no known algorithms for optimal approximation in the norm  $\|\cdot\|_\infty$ . However, the pioneering work of Adamjan et al (1971, 1978) has led to a theory of optimal approximation in the so-called Hankel norm. More recently, Glover (1984) has derived bounds on the  $L_\infty$ -norm of the approximation error  $B_t - M_a$ , where  $M_a$  is found using the optimal Hankel norm technique. The method is described next:

In the sequel, some assumptions are made concerning the plant  $P_t$  so that various technical assumptions are avoided. Task 2, discussed in the next section, is addressed to weakening some of these assumptions. Note that all of the assumptions below are automatically satisfied if  $P_t$  is rational, so that the problem at hand is one of approximating a finite-dimensional plant by another of lower McMillan degree.

(A1) The function  $P_t(s)$  has an unambiguous limit as  $|s| \rightarrow \infty$  with  $\text{Re } s \geq 0$ . Hence, we can speak about  $P_t(\infty)$ .

(A2)  $P_t$  has only isolated singularities in the closed RHP, and each of these is a pole of finite McMillan degree.

The above assumptions ensure (see Callier and Desoer 1978) that  $P_t$  has an n.r.c.f.  $B_t$ . Moreover, since  $P_t(\infty)$  is well-defined, so is  $B_t(\infty)$ . Now introduce the bilinear transform

$$z = \frac{s-1}{s+1}, \quad s = \frac{1+z}{1-z}. \quad (1.13)$$

Then the function  $B_t(s)$  gets transformed into an associated function  $B_t[(1+z)/(1-z)]$ , which we again denote (by a slightly sloppy notation) as

$B_t(z)$ . Note that when  $z=1$ ,  $s=\infty$ , so that  $B_t(\infty)$  must be well-defined in order for this procedure to be valid. Now  $B_t(z)$  belongs to the Hardy space  $H_\infty$  over the unit disk (see Duren 1970). Hence it has the power series

$$B_t(z) = \sum_{i=0}^{\infty} B_i z^i. \quad (1.14)$$

By the manner in which  $B_t$  was formed, it is always a "tall" matrix. Let us square up  $B_t$  as well as all of the  $B_i$ 's by adding columns of zeros, and form the block Hankel matrix

$$H = \begin{bmatrix} B_0 & B_1 & B_2 & \dots \\ B_1 & B_2 & \dots & \\ B_2 & \dots & & \\ \vdots & & & \\ \cdot & & & \end{bmatrix}. \quad (1.15)$$

Now  $H$  can be viewed as a bounded operator on the space  $\ell_2^n$ , consisting of all square summable,  $n$ -vector valued sequences. Moreover,  $H$  has finite rank if and only if  $B_t$  is rational. Now consider the matrix  $H^*H$  and examine its spectrum. If  $B_t$  is rational (i.e.  $P_t$  is lumped), then this spectrum consists solely of real eigenvalues. In general, if  $P_t$  has an infinite-dimensional state space, the spectrum is real but need not contain only eigenvalues. Suppose we wish to approximate  $P_t$  by a plant  $P_a$  of McMillan degree  $k$ . If it happens that the spectrum of  $H^*H$  consists of eigenvalues  $\sigma_0^2 \geq \sigma_1^2 \geq \dots \geq \sigma_k^2$ , plus a subset of the real line contained in  $[0, \sigma_k]$ , this is easily done. Suppose first that  $k > n$ , and that

$$\sigma_0^2 \geq \sigma_1^2 \geq \dots \geq \sigma_{k-n}^2 > \sigma_{k-n+1}^2 = \sigma_{k-n+2}^2 = \dots = \sigma_k^2. \quad (1.16)$$

In this case, there exist  $n$  linearly independent eigenvectors  $x^{(1)}, x^{(2)}, \dots, x^{(n)}$ , each belonging to  $\ell_2^n$ , of the matrix  $H^*H$ . That is, each vector  $x^{(1)}, \dots, x^{(n)}$  has the form

$$x^{(i)} = \{x_0^{(i)}, x_1^{(i)}, \dots\} \rightsquigarrow \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ \vdots \end{bmatrix} \quad (1.17)$$

Let  $y^{(i)}$  denote  $Hx^{(i)}$ , and partition each  $y^{(i)}$  as above. Finally, define the functions

$$x_i(z) = \sum_{j=0}^{\infty} x_j^{(i)} z^j, \quad X(z) = [x_1(z) \dots x_n(z)], \quad (1.18)$$

and note that  $X(z)$  is an  $n \times n$  matrix-valued function. Define  $y_i(z)$  and  $Y(z)$  accordingly. Then Adamjan et al. 1978 show that the function

$$M_a(z) = B_t - \sigma_k X(z)[Y(z)]^{-1} \quad (1.19)$$

has McMillan degree  $k$ . Moreover, Glover 1984 has shown that

$$\|B_t - M_a\|_{\infty} \leq 2 \sum_{i=k+1}^{\infty} \sigma_i, \quad (1.20)$$

if the spectrum of  $H^*H$  consists solely of eigenvalues. If the multiplicity condition (1.16) does not hold, then one has to use so-called augmentation (see Adamjan et al. 1978). We will not discuss it here in the interest of brevity.

So far we have accomplished the following: given a possibly distributed plant  $P_t$ , we have formed the associated inner matrix  $B_t$ , and found a

stable matrix  $M_a$  of McMillan degree  $k$  that approximates  $B_t$ . The last step is to go from the matrix  $M_a$  to the approximating plant  $P_a$ . This is done as follows: First, carry out the  $z$  to  $s$  transformation of (1.13), and partition  $M_a(s)$  in the form

$$M_a(s) = \begin{bmatrix} N_a(s) \\ D_a(s) \end{bmatrix}. \quad (1.21)$$

Then define

$$P_a(s) = N_a(s) [D_a(s)]^{-1}. \quad (1.22)$$

It can be shown that the McMillan degree of  $P_a$  is at most  $k$ , and is generically equal to  $k$ . An easy way to see this is as follows: Let

$(A_m, B_m, C_m, E_m)$  be a state-space realization of the transfer matrix  $M_a(s)$ , and partition

$$C_m = \begin{bmatrix} C_N \\ C_D \end{bmatrix}, \quad E_m = \begin{bmatrix} E_N \\ E_D \end{bmatrix} \quad (1.23)$$

Then, provided  $E_D$  is nonsingular, which it is generically, a state-space realization for  $P_a$  is given by the quadruple

$$\begin{aligned} A_a &= A_m - B_m E_D^{-1} C_D, \quad B_a = B_m, \\ C_a &= C_N - E_N E_D^{-1} C_D, \quad E_a = E_N E_D^{-1}. \end{aligned} \quad (1.24)$$

In summary, we have presented an order reduction procedure that has the following novel and desirable features:

- (i) It allows approximation of distributed plants by lumped plants.

- (ii) It allows one to specify a priori the McMillan degree of the approximate plant.
- (iii) It gives an a priori upper bound on the graph metric distance between the true and approximate plants. This bound, together with the stability condition (1.8), can be used to determine a priori whether a controller designed to stabilize the approximate plant also stabilizes the true plant.

One final feature of the approximation procedure is worth noting. Even if  $P_t$  has infinitely many unstable poles,  $P_a$  has only finitely many. Further, the locations of the unstable poles of  $P_a$  need not correspond to those of  $P_t$ . This feature defies conventional wisdom, which states that, when approximating an unstable plant, one must faithfully reproduce the "unstable part" of the plant. This is made possible by the use of the graph metric, which allows one to approximate the stable factors of a plant rather than the plant itself.

## 2. TASK 2: COPRIME FACTORIZATIONS

Task 2 was as follows: Extend the existing theory of designing stabilizing controllers for linear distributed systems to the case where the plant has an infinite number of right half-plane poles, with particular emphasis on deriving conditions for the existence of a right-coprime factorization in such a case.

The motivation behind this task is as follows: Vidyasagar 1978 showed that every rational matrix has both right and left coprime factorizations over  $\hat{A}$ . More generally, Callier and Desoer 1978 introduced a class that they call  $\hat{B}_\infty$ , with the property that every matrix whose elements all belong to  $\hat{B}_\infty$  has both an r.c.f. and an l.c.f. However, functions in  $\hat{B}_\infty$  can have only finitely many singularities in the closed RHP, and each of these must be a pole of finite order. Nett et al 1983 showed that plants in  $M(\hat{B}_\infty)$  could be stabilized by lumped compensators. The objective of this task is to see whether similar statements can be made about plants with infinitely many unstable poles (which, per force, cannot belong to  $\hat{B}_\infty$ ).

To motivate the discussion, consider the plant

$$p(s) = \tanh s, \quad (2.1)$$

which has poles and zeros alternating along the  $j\omega$ -axis. Hence this is an infinite analogy of an LC admittance, i.e. a nondissipative system. It is easy to see that  $p(s)$  has a coprime factorization over  $\hat{A}$ , since

$$p(s) = \frac{1 - e^{-2s}}{1 + e^{-2s}} \quad (2.2)$$

and since  $n(s)+d(s)=1$ . Hence it is certainly possible for plants with infinitely many unstable poles to have coprime factorizations. The objective thus is to determine necessary and sufficient conditions for a plant  $p$  to have a coprime factorization.

The question can be divided into two parts: (i) Given  $p(s)$ , when does it have a fractional representation, i.e. when can it be expressed as a ratio  $n(s)/d(s)$ , where  $n, d \in \mathcal{A}$ ?; (ii) Given a  $p(s)$  with a fractional representation, when does it have a coprime factorization?

The first question is very difficult to answer in very general terms, since the number of possible functions  $p(s)$  is limitless. However, a few quick necessary conditions can be used to rule out functions that don't have fractional representations.

Theorem 2.1 A function  $p(s)$  can be expressed as a ratio  $n(s)/d(s)$  where  $n, d \in \mathcal{A}$  only if it satisfies the following conditions:

- (i) In the open half-plane  $\text{Re } s > 0$ , the only singularities of  $p$  are poles of finite order.
- (ii) On any vertical line  $\sigma + j\omega$ ,  $\omega \in \mathbb{R}$ ,  $\sigma > 0$ , the zeros and poles of  $p$  are asymptotically almost periodic.

Proof Any function  $f \in \mathcal{A}$  has the form

$$f(s) = \sum_{i=0}^{\infty} f_i e^{-st} + f_a(s), \quad (2.3)$$

where  $f_a$  is the Laplace transform of an  $L_1$  function. Hence, by the Riemann-Lebesgue lemma,  $|f_a(s)| \rightarrow 0$  whenever  $|s| \rightarrow \infty$  with  $\text{Re } s > 0$ . Now the function

$$f_{ap}(j\omega) = \sum_{i=0}^{\infty} f_i e^{-j\omega t_i} \quad (2.4)$$

is almost-periodic, as shown by Callier and Desoer 1972. In fact, if the delays  $t_i$  are periodic, i.e. if  $t_i = iT$  for some constant  $T > 0$ , then  $f_{ap}$  is actually periodic. Hence, if  $p(s) = n(s)/d(s)$  and  $n, d \in \hat{A}$ , then along any vertical line  $\sigma + j\omega$  with  $\sigma > 0$ , the functions  $n(\sigma + j\omega)$  and  $d(\sigma + j\omega)$  converge respectively to their almost periodic parts. This proves (ii). To prove (i), note that  $d$  is analytic in the open RHP, and as a consequence its zeros are isolated and of finite multiplicity.

Using Theorem 2.1, one can quickly conclude that

$$p(s) = \frac{1}{\sqrt{s-1}} \quad (2.5)$$

does not have a fractional representation, since its singularity at  $s=1$  is a branch point.

The answer to the second question is, in principle, contained in the following result from Vidyasagar 1985, Section 8.1.

Lemma 2.2 Suppose  $p(s) = n(s)/d(s)$ , with  $n, d \in \hat{A}$ . Then  $p$  has a coprime factorization if and only if the ideal in  $\hat{A}$  generated by  $n$  and  $d$  is principal.

Though Lemma 2.2 contains a necessary as well as sufficient condition, it is difficult to apply it to a specific situation without further effort. The next result provides a useful sufficient condition.

Theorem 2.3 Suppose  $p(s)$  has a fractional representation  $n(s)/d(s)$  where  $n, d \in \hat{A}$ . Then  $p$  has a coprime factorization provided the number of RHP zeros of either  $n$  or  $d$  is finite.

Proof We consider the case where  $n$  has only finitely many zeros in the closed RHP, the other case follows by symmetry. Let  $z_1, \dots, z_k$  denote the RHP zeros of  $n$ , and assume, by renumbering if necessary, that  $z_1, \dots, z_r$  are also zeros of  $d$ , while the rest are not. Let  $m_1, \dots, m_r$  denote the minimum multiplicities of these as common zeros of  $n$  and  $d$ . Then, as shown by Callier and Desoer 1978, the function

$$f(s) = \prod_{i=1}^r \left[ \frac{s - z_i}{s + 1} \right]^{m_i} \quad (2.6)$$

divides both  $n$  and  $d$ . Let  $n_1 = n/m, d_1 = d/m$ . Then, by assumption,  $n_1$  and  $d_1$  have no common zeros, and  $n_1$  is bounded away from zero at infinity; that is, if  $\{s_i\}$  is any sequence in the closed RHP with  $|s_i| \rightarrow \infty$ , we have  $\liminf |n(s_i)| > 0$ . Hence

$$\inf_{\operatorname{Re} s \geq 0} |n_1(s)| + |d_1(s)| > 0. \quad (2.7)$$

This implies that  $n_1$  and  $d_1$  are coprime, as shown by Callier and Desoer 1978. Since clearly  $p = n/d = n_1/d_1$ , it follows that  $(n, d)$  is a coprime factorization of  $p$ .

We conclude this section by giving a very definitive theorem concerning coprime factorizations over the disk algebra  $A_D$ .

Theorem 2.4 Suppose  $p(z)$  has a fractional representation  $n(z)/d(z)$ , where  $n, d \in A_D$ . Then  $p$  has a coprime factorization over  $A_D$  if and only if the following condition holds: Let  $Z_n$  denote the set of zeros of  $n$  inside the closed unit disk, and define  $Z_d$  similarly. Then the cluster points of  $Z_n$  and  $Z_d$  do not intersect.

Before proceeding to the proof, let us explain what the theorem means. Recall that a point  $y$  is a cluster point of a set  $S$  if every neighborhood of  $y$  contains a point of  $S$  other than  $y$ . Also, note that since both  $n$  and  $d$  are analytic in the open unit disk, any zeros of these functions therein must be isolated. Hence the only cluster points of  $Z_n$  and  $Z_d$  (if any) must be on the unit circle. Thus the theorem says that  $n$  and  $d$  can have common zeros in the open unit disk, and they can have common zeros on the unit circle provided that any such point is an isolated zero of at least one of the functions. But there cannot be a point on the unit circle which is a limit of both a sequence of zeros of  $n$  and a sequence of zeros of  $d$ .

Proof "If" Suppose the hypothesis holds. Then there can only be finitely many common zeros of  $n$  and  $d$  in the closed unit disk, and each has only a finite multiplicity for at least one of the functions. Hence, as in the proof of Theorem 2.3, one can extract a polynomial as a common divisor of both functions.

"Only if" This part of the proof closely follows Vidyasagar et al, 1982. Suppose  $|z_0| = 1$ , and that  $z_0$  is the limit of a sequence  $\{a_i\}$  of

zeros of  $n$  as well as of a sequence  $\{b_i\}$  of zeros of  $d$ . We show that the ideal generated by  $n$  and  $d$  in  $A_D$  cannot be principal. Suppose by way of contradiction that there exist functions  $f, g, h, x, y$  in  $A_D$  such that

$$xn + yd = h, \quad n = fh, \quad d = gh. \quad (2.8)$$

Then  $gn = fd$ , so that  $f(a_i) = 0, g(b_i) = 0$ . Also, since

$$xf + yg = 1, \quad (2.9)$$

we have

$$x(b_i) f(b_i) = 1 \text{ for all } i \quad (2.10)$$

Now let  $i \rightarrow \infty$ . Then  $\lim b_i = \lim a_i = z_0$ . Since  $f$  is continuous at  $z_0$ , we have

$$f(z_0) = \lim_i f(a_i) = 0. \quad (2.11)$$

But (2.10) shows that

$$x(z_0) f(z_0) = \lim x(b_i) f(b_i) = 1. \quad (2.12)$$

This contradiction concludes the proof.

### 3. TASK 3: ROBUSTNESS

Task 3 was as follows: Investigate the simultaneous stabilizability of an infinite family of systems.

In its most abstract version, the problem is stated as follows in Vidyasagar 1985, Chapter 7. Let  $\Lambda$  be a first-countable topological space, let  $\lambda_0$  be an element of  $\Lambda$ , and let  $P_\lambda$ ,  $\lambda \in \Lambda$  be a family of plants. When does there exist a neighborhood  $B$  of  $\lambda_0$  such that there exists a common stabilizing controller for all plants  $p_\lambda$ ,  $\lambda \in B$ ? The explanation behind this problem statement is as follows:  $\lambda$  represents a set of physical parameters, and  $P_\lambda$  represents the corresponding system description.  $\lambda_0$  is the nominal set of parameter values, and  $p_{\lambda_0}$  is the corresponding nominal plant description. The objective is to determine whether a stabilizing controller for the nominal plant  $p_{\lambda_0}$  also stabilizes all plants  $p_\lambda$  when  $\lambda$  is "sufficiently close" to  $\lambda_0$ , i.e. whether there is a controller that achieves stability under nominal conditions and also maintains stability when the parameters are slightly perturbed.

In one sense, the above problem formulation is incomplete. The objective of controller design is not merely to stabilize the plant but also to achieve a desired closed-loop response. With this in mind, the following approach is used in Vidyasagar 1984, 1985. Define the closed-loop transfer matrix

$$H(P,C) = \begin{bmatrix} (I+PC)^{-1} & -P(I+CP)^{-1} \\ C(I+PC)^{-1} & (I+CP)^{-1} \end{bmatrix} \quad (3.1)$$

Say that the family  $\{P_\lambda\}$  is robustly stabilizable at  $\lambda_0$  if there exists a neighborhood  $B$  of  $\lambda_0$  and a controller  $C$  such that

- (i)  $C$  stabilizes  $P_\lambda$  for all  $\lambda$  in  $B$ , and
- (ii)  $H(P_\lambda, C)$  is continuous in  $\lambda$  at  $\lambda_0$ .

With this definition, a complete solution is given in Vidyasagar 1985, namely: robust stabilization is possible if and only if  $P_\lambda$  is continuous in the graph topology at  $\lambda_0$ . The graph topology is discussed at great length in this reference, and a characterization of continuity that is adequate for present purposes is this:  $P_\lambda$  is continuous if and only if there exists an r.c.f.  $(N_0, D_0)$  of  $P_\lambda$  and a family of r.c.f.'s  $(N_\lambda, D_\lambda)$  of  $P_\lambda$  such that  $N_\lambda \rightarrow N_0$ ,  $D_\lambda \rightarrow D_0$ .

In view of the aforementioned result, the only remaining interest in this problem concerns the case where the requirement that  $H(P_\lambda, C)$  be continuous is removed. Suppose  $(N_\lambda, D_\lambda)$  is an r.c.f. of  $P_\lambda$ , that  $N_\lambda, D_\lambda$  are continuous, and approach  $N_0, D_0$  respectively as  $\lambda \rightarrow \lambda_0$ . Let  $P_0 = N_0 D_0^{-1}$ . Is there a controller that stabilizes  $P_0$  as well as  $P_\lambda$  for all  $\lambda$  in some neighborhood of  $\lambda_0$ ? If  $N_0$  and  $D_0$  are right-coprime, the answer is clearly "yes", by virtue of the aforesaid results. Consider now the case where  $N_0, D_0$  are not coprime, so that there is pole-zero cancellation in the extended closed RHP. One can ask: Is it still possible to stabilize simultaneously  $P_0$  as well as  $P_\lambda$ ? The next theorem shows that this is possible only if the cancellation is on the  $j\omega$ -axis or at infinity.

Theorem 3.1 Suppose  $N_\lambda, D_\lambda$  are continuous with limits  $N_0, D_0$ , respectively as  $\lambda \rightarrow \lambda_0$ . Suppose  $N_\lambda, D_\lambda$  are right-coprime for  $\lambda \neq \lambda_0$  and let  $R$  be a greatest common right divisor of  $N_0, D_0$ . Let  $r = |R|$ ,  $P_0 = N_0 D_0^{-1}$ ,  $P_\lambda = N_\lambda D_\lambda^{-1}$ . If  $r$  has any zeros in the open RHP, then there does not exist a controller that stabilizes both  $P_0$  and  $P_\lambda$ ,  $\lambda \neq \lambda_0$ .

Proof We prove the contrapositive. Suppose  $C$  stabilizes both  $P_0$  and  $P_\lambda$  for  $\lambda \neq \lambda_0$ . Let  $N_0 = NR$ ,  $D_0 = DR$ . Then  $N, D$  are right-coprime, and there exists a l.c.f.  $(\overline{D_C}, \overline{N_C})$  of  $C$  such that

$$\overline{D_C} D + \overline{N_C} N = I. \quad (3.2)$$

Now consider the return difference matrix

$$U_\lambda = \overline{D_C} D_\lambda + \overline{N_C} N_\lambda. \quad (3.3)$$

Since  $C$  stabilizes  $P_\lambda$ ,  $U_\lambda$  is unimodular for all  $\lambda \neq \lambda_0$ . Letting  $\lambda \rightarrow \lambda_0$  in (3.3) gives

$$R = \lim_{\lambda \rightarrow \lambda_0} U_\lambda. \quad (3.4)$$

In particular,  $r = |R|$  is the limit of  $|U_\lambda|$ . Now  $|U_\lambda|$  has all of its zeros in the open LHP, since it is a unit. Hence the zeros of  $r$  can at best be on the  $j\omega$ -axis or at infinity.

As an application of Theorem 3.1, suppose

$$p_\lambda(s) = \frac{s-1-\lambda}{s-1+\lambda}, \quad \lambda_0 = 0, \quad p_0(s) = 1 \quad (3.5)$$

Then no controller can stabilize both  $P_0$  and  $P_\lambda$ ,  $\lambda \neq 0$ .

Theorem 3.1 is the best possible result, because if  $|R|$  does not have open RHP zeros, then it may be possible to achieve simultaneous stabilization. This is illustrated by the following singular perturbations example.

Let  $\Lambda = [0, \infty]$ ,  $\lambda_0 = 0$ , and consider the plants  $p_\lambda, p_0$  described in state-space form by the equations

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda z} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u, \quad y = x + z + 2u, \quad (p_\lambda)$$

$$\dot{x} = 3x - u, \quad y = 2x + u, \quad (p_0)$$

or in the transfer function form by

$$p_\lambda(s) = \frac{2\lambda s^2 + (1-\lambda)s - 5}{\lambda s^2 + (1-\lambda)s - 3}, \quad p_0(s) = \frac{s-5}{s-3}.$$

Now a coprime factorization of  $p_\lambda$  is given by

$$n_\lambda(s) = \frac{2\lambda s^2 + (1-\lambda)s - 5}{(s+1)^2}, \quad d_\lambda(s) = \frac{\lambda s^2 + (1-\lambda)s - 3}{(s+1)^2}.$$

As  $\lambda \rightarrow 0$ , we get

$$n_{\lambda \rightarrow 0} = \frac{s-5}{(s+1)^2}, \quad d_{\lambda \rightarrow 0} = \frac{s-3}{(s+1)^2}.$$

Clearly  $n_0$  and  $d_0$  are not coprime, since they have the nominal common divisor

$$r(s) = \frac{1}{s+1}.$$

However, the controller

$$c(s) = -\frac{8}{s+12}$$

stabilizes  $p_0$  as well as  $p_\lambda$  for sufficiently small positive values of  $\lambda$ .

In this case the only zero of the common factor  $r$  is at infinity.

#### 4. TASK 4: FLEXIBLE STRUCTURE EXAMPLE

In order to illustrate the usefulness of the Stable Factorization approach in a specific control example, a four-disk model of a flexible structure will be presented here.

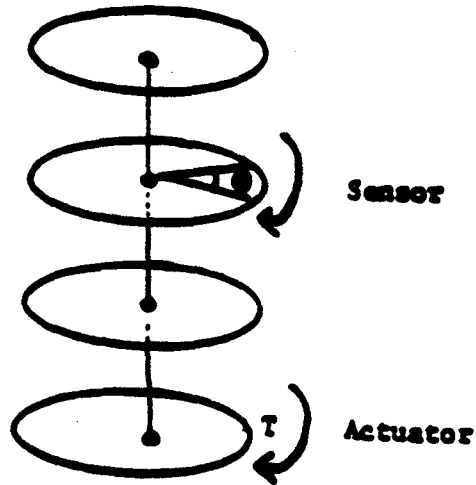


Figure 1

The disks are connected by torsion springs. Let  $K_i$  denote the spring constant for the spring between  $i$ -th and  $(i+1)$ -th disk; let  $J_i$  denote the inertia for  $i$ -th disk. The equations of motion for the system are:

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} + \begin{bmatrix} \frac{K_1}{J_1} & -\frac{K_1}{J_1} & 0 & 0 \\ \frac{K_1}{J_2} & +\frac{(K_1 + K_2)}{J_2} & \frac{K_2}{J_2} & 0 \\ 0 & -\frac{K_2}{J_3} & \frac{+(K_2 + K_3)}{J_3} & -K_3 \\ 0 & 0 & -\frac{K_3}{J_4} & \frac{K_3}{J_3} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{T_3}{J_3} \\ 0 \end{bmatrix} \quad (S)$$

where  $\theta_i$  denotes the angle of deflection of the  $i$ -th disk from normal position. Each dot indicates differentiation with respect to time. The right hand side indicates the scaled torques applied (in this case to the 3rd disk).

The motivation of this model is that by applying actuator inputs (torques)  $u_i$  and disk  $i$ , one wishes to control the outputs  $\theta_j$ , for  $i$  equal to or different from  $j$ . The transfer function from each input  $u_i$  to each output  $\theta_j$  can be easily calculated from the dynamic system (S). For the purpose of illustration we assume that the spring constants  $K_i$  are equal to unity. The nominal plant is further assumed to have  $J_i = 1$ ,  $i=1, 2, 3, 4$  (e.g. equal inertia). Note that due to the fact that there is no damping in the system, the transfer function from any input to any output will have all of its poles and zeros located on the  $j\omega$ -axis.

For example, the transfer function from  $u_3$  to  $\theta_4$  will be of the form

$$P(s) = \frac{b(s)}{a(s)}$$

$$= \frac{s^2 + \beta_1^2}{s^2 \prod_{i=1}^4 (s^2 + \alpha_i^2)}$$

where  $\beta_1$  and  $\alpha_i$  are real positive scalars, corresponding to a zero at  $z_1 = j\beta_1$  and poles  $p_i = j\alpha_i$ ,  $i = 1, 2, 3, 4$ . The rigid body modes correspond to the poles  $s^2 = 0$ .

In the following we present a fixed controller that simultaneously stabilizes a nominal plant  $P_1$  and a contingent plant  $P_2$ . This is an example of the simultaneous stabilization techniques introduced in Vidyasagar and Viswanadham (1982) as discussed earlier. The particular pole zero locations correspond to the experimental set-up described in Cannon and Rosenthal (1984), also reported in Razavi, Mehra, and Vidyasagar (1985). The computation of the actual controller using state-space versions of the stable factorization approach is carried out based on methods of Minto 1985.

The system under study consists of four steel disks connected by three steel rods as shown in Figure 1. A control input is applied to the second disk from the top, and the two outputs are the positions of the first and fourth disks, respectively. Two different situations are considered. In the first, all four disks are identical. In the second, the first three disks are identical and the last disk has a moment of inertia that is one

quarter of those of the rest. In the nominal case, the system is denoted by  $p_1(s)$  and equals

$$p_1(s) = \frac{827,200}{a_1(s)} \begin{bmatrix} b_{11}(s) \\ b_{12}(s) \end{bmatrix}, \quad (4.1)$$

where

$$\begin{aligned} a_1(s) &= s^2 (s \pm j23.81) (s \pm j18.22) (s \pm j9.863), \\ b_{11}(s) &= (s \pm j20.85) (s \pm j7.96) \\ b_{12}(s) &= (s \pm j12.886) \end{aligned} \quad (4.2)$$

When the last disk is smaller than the rest, the system is

$$p_2(s) = \frac{827,200}{a_2(s)} \begin{bmatrix} b_{21}(s) \\ b_{22}(s) \end{bmatrix}, \quad (4.3)$$

where

$$\begin{aligned} a_2(s) &= s^2 (s \pm j29.34) (s \pm j21.48) (s \pm j11.938), \\ b_{21}(s) &= b_{11}(s), \\ b_{22}(s) &= (s \pm j25.95). \end{aligned} \quad (4.4)$$

The objective is to find a single controller  $C$  that stabilizes both  $p_1$  and  $p_2$ . In the present instance both systems are entirely undamped, i.e. all poles and zeros are purely imaginary. If we stabilize the two systems by moving the poles into the left half-plane, it is possible to come up with designs that yield closed-loop systems that are nominally stable, but have poles with very little damping. To rule out this possibility, we

define the stability region to be the region  $\{s: \operatorname{Re} s < -1\}$ . Thus, in order for the closed-loop system to be stable in this more restrictive sense, its poles must have real parts less than  $-1$ .

Using the methods of Minto 1985, we have designed a seventeenth order controller that achieves the above objectives. The closed-loop poles of the two compensated plants are shown in Table 1. The controller itself is shown in modal form in Appendix 1. Note that modes 7,8,11, and 15 can essentially be ignored, resulting in a thirteenth order controller.

(p<sub>1</sub>)(p<sub>2</sub>)

-1. 0011D+00	+2. 2294D+01i	-4. 0395D-01	+5. 9562D+00i
-9. 9991D-01	+2. 0850D+01i	-1. 0000D+01	+1. 2997D-12i
-1. 0011D+00	-2. 2294D+01i	-1. 7616D+00	-8. 5941D-01i
-9. 9991D-01	-2. 0850D+01i	-1. 7616D+00	+8. 5941D-01i
-1. 0079D+00	+1. 2837D+01i	-1. 5015D+00	+6. 8560D-10i
-1. 0079D+00	-1. 2837D+01i	-2. 4975D+00	-1. 3710D-09i
-2. 0000D+01	+2. 1744D-16i	-3. 4988D+00	-3. 2865D-09i
-9. 9952D-01	+7. 9635D+00i	-4. 5034D+00	+4. 4246D-09i
-9. 9952D-01	-7. 9635D+00i	-5. 4976D+00	-2. 2343D-09i
-9. 8470D-01	+2. 1881D-10i	-6. 5007D+00	+4. 8283D-10i
-1. 7303D+00	+8. 4607D-01i	-7. 4999D+00	-3. 9234D-11i
-1. 7303D+00	-8. 4607D-01i	-8. 5000D+00	+6. 4947D-13i
-2. 2062D+00	-3. 3423D-09i	-2. 9694D+01	-1. 1515D+01i
-2. 8208D+00	+2. 3092D-09i	-2. 9694D+01	+1. 1515D+01i
-4. 0676D+00	+2. 2116D-10i	-2. 9017D+00	-2. 7565D+01i
-4. 9849D+00	-7. 8559D-10i	-2. 9017D+00	+2. 7565D+01i
-9. 9938D+00	+5. 3563D-10i	-1. 0011D+00	-2. 2294D+01i
-7. 0059D+00	-1. 9077D-10i	-9. 9991D-01	-2. 0850D+01i
-7. 9971D+00	+3. 2835D-11i	-1. 0011D+00	+2. 2294D+01i
-1. 0307D+01	-4. 1801D-10i	-9. 9991D-01	+2. 0850D+01i
-1. 0109D+01	+2. 9323D-01i	-1. 0079D+00	-1. 2837D+01i
-1. 0109D+01	-2. 9323D-01i	-1. 0079D+00	+1. 2837D+01i
-9. 7368D+00	+2. 1002D-01i	-9. 9952D-01	-7. 9635D+00i
-1. 0002D+01	+1. 4719D-10i	-9. 9952D-01	+7. 9635D+00i
-9. 7368D+00	-2. 1002D-01i	-4. 0395D-01	-5. 9562D+00i

Table 1. Closed loop poles of nominal (p<sub>1</sub>) and contingent (p<sub>2</sub>) plants

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## APPENDIX 1

### Controller Equations for Simultaneous Stabilization (Section 4)

Controller dynamic equations:

$$\frac{d}{dt} x_c(t) = (AC)x_c(t) + (BC)u_c(t)$$

$$y_c(t) = (CC)x_c(t) + (EC)u_c(t)$$

Controller input  $u_c$ : 2x1 vector

output  $y_c$ : 1x1 scalar

state  $x_c$ : 17x1 vector

Controller matrices:

AC: 17x17 matrix (diagonal)

BC: 17x2 matrix

CC: 1x17 vector

EC: 1x2 vector

AC = DC - I<sub>17</sub>

I<sub>17</sub>: identity matrix of dimension 17

Diagonal elements of DC =

```

-3. 1626D+01 +1. 9655D+011
-3. 1626D+01 -1. 9655D+011
-1. 9587D+00 +2. 7792D+011
-1. 9587D+00 -2. 7792D+011
 1. 4405D+01 +1. 1825D+011
 1. 4405D+01 -1. 1825D+011
 9. 2047D-01 +7. 8696D+001
 9. 2047D-01 -7. 8696D+001
-1. 8641D+01 +4. 4488D+001
-1. 8641D+01 -4. 4488D+001
-7. 5467D+00 +3. 3470D+001
-7. 5467D+00 -3. 3470D+001
-9. 3310D+00 +1. 5756D+001
-9. 3310D+00 -1. 5756D+001
-1. 0000D+01 +3. 6736D-141
-8. 9921D+00 +4. 1208D-011
-8. 9921D+00 -4. 1208D-011

```

BC =

```

-4. 7854D-02 -1. 0220D-021
-1. 0925D+01 +2. 1710D+011
 1. 7011D+01 +4. 6269D+001
-1. 2311D-03 +7. 1597D-041
-7. 5782D-05 +7. 1595D-041
-1. 1869D+01 -3. 2638D+011
 2. 1277D+01 +6. 2140D+011
 2. 1589D-05 -2. 2810D-051
-8. 6212D-02 +2. 4038D-021
-1. 0469D-01 -6. 5944D-021
 4. 7822D-02 +7. 2364D-021
-9. 8152D-09 +3. 7046D-071
 3. 9727D+00 +5. 5967D+001
-7. 4894D-01 -4. 8371D-011
 9. 4173D+01 +1. 6482D+021
 3. 6119D+00 -1. 2755D+001
-3. 9984D-02 -3. 7931D-021

```

CC =

Columns 1 to 3  
-5.9795D+01 -1.5807D+01 -1.7310D+03 +2.5294D+03 -9.3229D+02 +8.7858D+03

Columns 4 to 6  
-8.7537D+03 -4.7325D+03 2.4804D+00 +1.3435D+01 -1.6045D+01 +9.8320D+00

Columns 7 to 9  
1.3050D+01 -5.2722D+00 -2.0074D+01 +1.9291D+01 3.2937D+01 -6.2577D+01

Columns 10 to 12  
-7.2154D+00 +3.6489D+01 -1.0714D+04 -1.3412D+03 2.1725D+00 +3.1051D+00

Columns 13 to 15  
4.6407D+01 -2.8844D+01 -4.2587D+01 +6.5548D+01 -1.8345D+02 -5.4209D+01

Columns 16 to 17  
-1.1577D+01 +3.0041D+01 2.3494D+00 -7.8900D+01

EC =

-4.4738D-01 0.0000D+00

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16. Abstract  Large space structures are characterized by the features that (i) they are in general infinite-dimensional systems, and (ii) they have large numbers of undamped or lightly damped poles. Any attempt to apply linear control theory to large space structures must therefore take into account these features. Phase I of this project consisted of an attempt to apply the recently developed Stable Factorization (SF) design philosophy to problems of large space structures, with particular attention to the aspects of robustness and fault tolerance.  The final report on the Phase I effort consists of four sections, each devoted to one task. The first three sections report new theoretical results, while the last consists of a design example. Significant new results were obtained in all four tasks of the project. More specifically, (a) an innovative approach to order reduction was obtained, (b) stabilizing controller structures for plants with an infinite number of unstable poles were determined under some conditions, (c) conditions for simultaneous stabilizability of an infinite number of plants were explored, and (d) a fault-tolerant controller design that stabilizes a flexible structure model was obtained which is robust against one failure condition.					
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